



Cambridge International AS & A Level

CANDIDATE
NAME

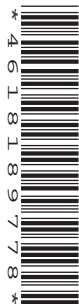
--

CENTRE
NUMBER

--	--	--	--	--

CANDIDATE
NUMBER

--	--	--	--



FURTHER MATHEMATICS

9231/31

Paper 3 Further Mechanics

October/November 2020

1 hour 30 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

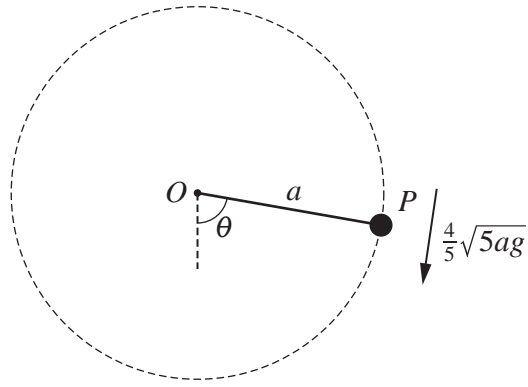
INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.
- Where a numerical value for the acceleration due to gravity (g) is needed, use 10 ms^{-2} .

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [].

This document has **16** pages. Blank pages are indicated.



A particle P is attached to one end of a light inextensible string of length a . The other end of the string is attached to a fixed point O . The particle P is held with the string taut and making an angle θ with the downward vertical. The particle P is then projected with speed $\frac{4}{5}\sqrt{5ag}$ perpendicular to the string and just completes a vertical circle (see diagram).

Find the value of $\cos \theta$. [5]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

- 3 One end of a light elastic string, of natural length a and modulus of elasticity $4mg$, is attached to a fixed point O . The other end of the string is attached to a particle of mass m . The particle moves in a horizontal circle with a constant angular speed $\sqrt{\frac{g}{a}}$ with the string inclined at an angle θ to the downward vertical through O . The length of the string during this motion is $(k+1)a$.

(a) Find the value of k .

[4]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

(b) Find the value of $\cos \theta$.

[2]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

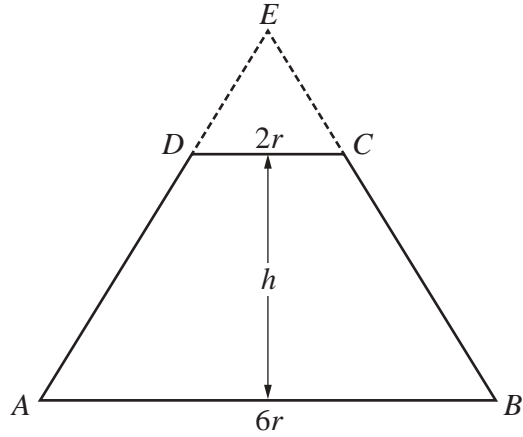
.....

.....

.....

.....

.....



The diagram shows the cross-section $ABCD$ of a uniform solid object which is formed by removing a cone with cross-section DCE from the top of a larger cone with cross-section ABE . The perpendicular distance between AB and DC is h , the diameter AB is $6r$ and the diameter DC is $2r$.

- (a) Find an expression, in terms of h , for the distance of the centre of mass of the solid object from AB . [4]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

The object is freely suspended from the point B and hangs in equilibrium. The angle between AB and the downward vertical through B is θ .

- (b) Given that $h = \frac{13}{4}r$, find the value of $\tan \theta$. [2]

.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....

5 A particle P is projected with speed u at an angle α above the horizontal from a point O on a horizontal plane and moves freely under gravity. The horizontal and vertical displacements of P from O at a subsequent time t are denoted by x and y respectively.

(a) Derive the equation of the trajectory of P in the form

$$y = x \tan \alpha - \frac{gx^2}{2u^2} \sec^2 \alpha. \quad [3]$$

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

The point Q is the highest point on the trajectory of P in the case where $\alpha = 45^\circ$.

(b) Show that the x -coordinate of Q is $\frac{u^2}{2g}$. [3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

(c) Find the other value of α for which P would pass through the point Q . [4]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

6 Two smooth spheres A and B have equal radii and masses m and $2m$ respectively. Sphere B is at rest on a smooth horizontal floor. Sphere A is moving on the floor with velocity u and collides directly with B . The coefficient of restitution between the spheres is e .

(a) Find, in terms of u and e , the velocities of A and B after the collision. [3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

Subsequently, B collides with a fixed vertical wall which makes an angle θ with the direction of motion of B , where $\tan \theta = \frac{3}{4}$.

The coefficient of restitution between B and the wall is $\frac{2}{3}$. Immediately after B collides with the wall, the kinetic energy of A is $\frac{5}{32}$ of the kinetic energy of B .

(b) Find the possible values of e . [7]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

A series of 20 horizontal dotted lines for writing.

- 7 A particle P moving in a straight line has displacement x m from a fixed point O on the line at time t s. The acceleration of P , in ms^{-2} , is given by $\frac{200}{x^2} - \frac{100}{x^3}$ for $x > 0$. When $t = 0$, $x = 1$ and P has velocity 10ms^{-1} directed towards O .

(a) Show that the velocity $v \text{ms}^{-1}$ of P is given by $v = \frac{10(1-2x)}{x}$. [5]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

Dotted lines for writing

BLANK PAGE

Permission to reproduce items where third-party owned material protected by copyright is included has been sought and cleared where possible. Every reasonable effort has been made by the publisher (UCLES) to trace copyright holders, but if any items requiring clearance have unwittingly been included, the publisher will be pleased to make amends at the earliest possible opportunity.

To avoid the issue of disclosure of answer-related information to candidates, all copyright acknowledgements are reproduced online in the Cambridge Assessment International Education Copyright Acknowledgements Booklet. This is produced for each series of examinations and is freely available to download at www.cambridgeinternational.org after the live examination series.

Cambridge Assessment International Education is part of the Cambridge Assessment Group. Cambridge Assessment is the brand name of the University of Cambridge Local Examinations Syndicate (UCLES), which itself is a department of the University of Cambridge.